Addition Rings — *E006 scoring rubric*

Math Domain

☑ Number/Quantity	☐ Shape/Space	☑ Function/Pattern
☐ Chance/Data	☐ Arrangement	

Math Actions (possible weights: 0 through 4)

2 Modeling/Formulating	2 Manipulating/Transforming
2 Inferring/Drawing Conclusions	2 Communicating

Math Big Ideas

☐ Scale	☐ Reference Frame	☑ Representation
☐ Continuity	☐ Boundedness	☐ Invariance/Symmetry
☑ Equivalence	☐ General/Particular	☐ Contradiction
☐ Use of Limits	☐ Approximation	☐ Other

The intent of this problem is to have students demonstrate arithmetic computation skills, specifically the relation between addition and its inverse.

No matter what number is put into the top slot in **1** and **2**, going through the other three and returning to the top produces the same number.

> It might be interesting to note that going clockwise through +3 and –1 and going counterclockwise through +2 gives the same result. Second possibly interesting thing to note is that the **final** result does not depend on the direction taken — both result in the net change of 0.

However, in **1** and **2,** if the original number is 1, 2 or 0, the number at the bottom circle (if moving counterclockwise) turns out to be negative and may cause some anxiety. To avoid this difficulty a larger number should be chosen.

It turns out, that starting from the bottom circle in **3** (with a number greater than 3) does not change the general result: going around the full circle gets you back to the original number (independent of the direction).

These results are all due to the fact that the operations are balanced or counteracted by each other — the combined effect of all the additions is counteracted by the net effect of all the subtractions. In other words, adding 2 and 3 is the same as adding 1 and 4, so adding 2 and 3 and then subtracting 1 and 4 is the same as adding 1 and 4 and subtracting 1 and 4. The operations of adding a number and subtracting the same number have inverse or opposite effect, so the net result is no change.

If students are having a difficult time in explaining the results in **4**, they may find it helpful to go through the process of designing their own ring in **5**.

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	partial level	full level
Modeling/ Formulating (weight: 2)	In **5**, design a ring that may not “work” in the sense of adding and subtracting equal amounts, or that uses the same numbers as the original ring in a different order.	In **5**, design a ring that is entirely correct and uses different numbers from the original ring.
Transforming/ Manipulating (weight: 2)	Get a correct result for all but one of questions **1-3**.	Get a correct result for all questions and be able to adapt if the initial number is too small, causing a negative number at other points in the ring.
Inferring/ Drawing Conclusions (weight: 2)	Recognize that the result does not depend on the starting point in the ring, Get the correct answer in **3**. but not able to articulate the reason in **4**. Design a ring in **5** that is little more than a copy of the given ring.	Articulate the conclusion with respect to the initial position in the ring and provide at least a rudimentary generalization of results in **3**. Exhibit an understanding of the process described by the ring, both through a clear verbal answer to **4**, and a ring design in **5** which uses completely different numbers and sequence of addition and subtraction. Extra: note that the effect does not depend on the direction of calculations in the ring, or the initial position.
Communicating (weight: 2)	Present evidence in **1–3** by exhibiting some calculations or intervening answers, without a clear verbalization of the final result. Give a partial or unclear explanation for **4.**	Give a full, clear explanation for all questions.