Greater, Lesser, In-Between *M020 scoring rubric*

Math Domain

☑ Number/Quantity	☐ Shape/Space	☐ Function/Pattern
☐ Chance/Data	☐ Arrangement	

Math Actions (possible weights: 0 through 4)

0 Modeling/Formulating	2 Manipulating/Transforming
3 Inferring/Drawing Conclusions	0 Communicating

Math Big Ideas

☐ Scale	☐ Reference Frame	☐ Representation
☑ Continuity	☐ Boundedness	☐ Invariance/Symmetry
☐ Equivalence	☐ General/Particular	☐ Contradiction
☐ Use of Limits	☐ Approximation	☐ Other

Pre-Activity

Since the first four digits are the same (0.239) in both numbers, the greater number must have the next digit greater than the smaller number. For example, 0.2394 would be greater than 0.2392 or 0.2393. Similarly, as the first three digits coincide in the second pair of numbers, the greater number must have the fourth digit greater, e.g. 11.295. The actual digits do not matter, as long as the relationship is maintained.

For fractions with the same denominator, the greater the numerator, the greater the fraction. Any pair satisfying this relationship, e.g. 3/8 vs. 1/8, would work. On the other hand, since the top fraction of 9/_ vs. 8/_ must be greater, even having the same denominator would satisfy the conditions, e.g. 9/8 vs. 8/8. If the second denominator is greater, the corresponding fraction will be even smaller.

Comparison of decimal fractions is done from left to right, so the digits must be selected from left to right in such a manner that the first digit that is different from either the top or the bottom number must be no less than the corresponding digit in the lower number and no greater than the corresponding digit in the upper number. Reasonable choices for the first intermediate number are .3000, .3099, .31273, etc. For the second intermediate number, reasonable choices include all the choices that can be used for the first number, and all numbers with the first two digits 29, e.g. .2969, .2970, …, to .2999.

Task

1. a. 3.2_0 and 3.2_9 : as long as the missing digit in the top number is greater than the missing digit in the bottom number, the condition of the problem is satisfied.

3._1_ and 3.2__ : there are several possibilities. First, the first missing digit is 3 or greater, so the remaining digits can take on any value without changing the outcome. Second, the first missing digit in the top number is 2 and the second digit in the bottom number is 1; therefore the last digit of the top number must be greater than the last digit of the bottom number. Third, the first missing digit on top is 2, but the first missing digit

on the bottom is 0, so the remaining two missing digits can take on any values without affecting the result.

_._36 and _._35 : two combinations are possible. First, if the first digit of the top number is greater than the first digit of the bottom number, or if the first digits are the same and the second top digit is greater, the condition is satisfied. On the other hand, both pairs of digits can be identical and still satisfy the condition, e.g., 1.336 > 1.335 .

b. 3.2_0 and 3.2_9 : in this case, the missing digits can be the same, or the bottom missing digit must be greater.

3._1_ and 3.2__ : again, there several possibilities. If the first three digits of each number are 3.21, than the bottom number must have the fourth digit greater. If only the first two digits are the same (3.2) then the first missing digit of the bottom number should be greater than 1. Finally, if the first missing digit of the top number is 0 or 1, the remaining digits can be filled in any manner.

_._36 and _._35 —here, it is not enough for the pairs of digits to be the same. The first digit of the bottom number must be greater, or, if the first digits are the same, the second digit of the bottom number must be greater.

2. a. _/7 and 4/5 : the top fraction is greater if the missing digit is 6, 7, 8 or 9.

1_/30 and 5/8: the first fraction is greater if the numerator is 19 or greater, so the missing digit must be 9.

b. _/7 and 4/5: any digit not included in the solution of 2a is a possible solution of 2b, i.e., 0, 1, 2, 3, 4 or 5.

1_/30 and 5/8 : the same principle applies and any digit between 0 and 8 is allowed.

3. The number in the middle is always *smaller* than the top number. So the missing digits must represent something greater than 68, i.e., 69, 70, 71, etc.

In the second case, any number from the first set would satisfy the conditions, no matter what digits are missing in the top and the bottom numbers. However, other digits may be filled in.

4. a. Several strategies will lead to the solution. One possibility is to replace each fraction with an equivalent one whose denominator is double the original denominator. So, 9/13 = 18/26 < 19/26 < 20/26 = 10/13.

b. A similar strategy will work, but one must be more careful, e.g. 26/20 < 26/19 < 26/18. Another strategy is to find a common denominator and take the mean. If the numerators of the two fractions with the common denominator differ only by 1, double the denominators (and the numerators)., then pick a number in between.

c. Again, doubling the denominators and numerators will work: 9/13 = 90000/130000 = 180000/260000 < 180001/260000 < 180002/260000 =90001/130000.

	partial level (1 or 2)	full level (3)
Modeling/ Formulating (weight: 0)		
Transforming/ Manipulating (weight: 2)	Student handles some, but not all, of the place value and other numerical issues correctly.	Students handles all number and quantity issues correctly.
Inferring/ Drawing Conclusions (weight: 3)	Student is able to infer some, but not all, of the relationships correctly.	Student is able to infer the relationships correctly, although there may be minor representational errors.
Communicating (weight: 0)		