Pizza Toppings *HL074 scoring rubric*

Math Domain

- [ ] Number/Quantity
- [ ] Shape/Space
- [ ] Function/Pattern
- [ ] Chance/Data
- [x] Arrangement

Math Actions (possible weights: 0 through 4)

- 3 Modeling/Formulating
- 3 Manipulating/Transforming
- 2 Inferring/Drawing Conclusions
- 2 Communicating

Math Big Ideas

- [ ] Scale
- [ ] Reference Frame
- [ ] Representation
- [ ] Continuity
- [ ] Boundedness
- [ ] Invariance/Symmetry
- [ ] Equivalence
- [x] General/Particular
- [ ] Contradiction
- [ ] Use of Limits
- [ ] Approximation
- [x] Other

General combinatorial formulas for the numbers of combinations could provide a shortcut here, but they are not necessary to the solution of the problem. A direct computation and explanation would be adequate. Below, each "formula" is derived directly from the data.

1. The \$7.50 pizza has two toppings $(\$7.50 = \$6.00 + \mathbf{2} \cdot \$0.75)$, so the first topping can be chosen from among 14 toppings and the second from the remaining 13. Furthermore, the mushroom and anchovy pizza is identical to anchovy and mushroom, so the actual number of combinations is half the product of the numbers of possible toppings: $\frac{14 \cdot 13}{2} = 91$ pizzas.

2. It is possible to find the number of possible pizzas with 1, 2, ..., 14 different toppings plus a pizza with no toppings (if it is a "possible combination of toppings"). Then the total number of all possible pizzas is the sum of all of these. This computation is long and messy, which increases the chances of making a miscalculation.

On the other hand, consider counting pizzas in a different way. Make a list of toppings and for each pizza check off the toppings that are on it. Each topping can either be checked off or not checked off — two possibilities, so the total number of combinations of toppings (and thus pizzas) is $\underbrace{2 \cdot 2 \cdot \ldots \cdot 2}_{\text{14 terms}} = 2^{14} = 16384$.

3. Once again, taking the number of 0, 1, 2, ..., 14 topping pizzas and multiplying it by the cost of such a pizza is rather drawn out and tedious. On the other hand, let us count pizzas in pairs in the following way: for each two pizzas we use up all the toppings, but if the toppings are found on one pizza they are not on the other. This method counts all the pizzas exactly once. Then the total number of toppings is half the product of the number of pizzas and the number of topping selections, so the total cost is $16384 \cdot \$6.00 + \frac{16384}{2} \cdot 14 \cdot \$0.75 = \$184{,}320.00$.

With a 15% tip, that adds up to \$211,968.00.

4. There are multiple possibilities here. The factors to be determined are how many students are in the school (which will vary widely from school to school) and the amount of pizza each student eats. The range of plausible numbers is not particularly wide — anywhere between 1 and 8 students per pizza. For example, let us assume that there are 1000 students in the school and each student, on average, eats quarter of a pizza per day. The supply of pizzas would be finished in about 66 days. Since we are only talking about school days, this is nearly one third of a school year. Other assumptions will, of course, give different time periods.

5. There is a great variety of plausible answers here as well. One property that all reasonable graphs must have in common is two distinct intervals: first, the slice is heated up rather rapidly, which is indicated by a relatively sharp upward climb in temperature; the second interval takes up all the remaining time (a considerably longer period) and should generally have a shallower slope. The two could possibly be connected by a short horizontal stretch, indicating that the pizza is baking at constant temperature for a little while, but it is not necessary. The second interval may be composed of several straight or curved pieces of unequal slope, but each must show declining temperature and each should be accompanied by a reasonable explanation.

	partial level	full level
Modeling/ Formulating (weight: 3)	Devise a correct counting scheme for at least **1** or **2**. Sketch a graph in **5** that clearly shows rapid heating and gradual cooling.	Devise correct scoring schemes for both **1** and **2**. Give additional detailed information in the graph. Provide reasonable assumptions to allow for an estimate in **4**.
Transforming/ Manipulating (weight: 3)	Correctly evaluate the expressions in **1** and **2** and complete at least a part of the calculations in **3** and **4**.	Perform appropriate calculations in **3** and **4**. Must arrive at answers in **1-4**
Inferring/ Drawing Conclusions (weight: 2)	Develop a correct procedure for calculations in **3**.	Develop efficient counting procedures in **2** and **3**. Provide a reasonable estimate in **4**.
Communicating (weight: 2)	Display all calculations and state direct answers to the questions. In **5**, provide some graph showing temperature as a function of time.	Provide coherent explanations and state all assumptions where required, especially in **3** and **4**. Produce a fully annotated and labeled graph in **5**.