

**Math Domain**

- |   |                                      |  |
|---|--------------------------------------|--|
| <input checked="" type="checkbox"/> Number/Quantity | <input type="checkbox"/> Shape/Space | <input checked="" type="checkbox"/> Function |
| <input type="checkbox"/> Chance/Data                | <input type="checkbox"/> Arrangement |  |

**Math Actions** (possible weights: 0 through 4)

- |  |  |
|--|--|
| <input type="checkbox"/> 2 Modeling/Formulating          | <input type="checkbox"/> 2 Manipulating/Transforming |
| <input type="checkbox"/> 3 Inferring/Drawing Conclusions | <input type="checkbox"/> 2 Communicating             |

**Math Big Ideas**

- |  |   |  |
|--|---|--|
| <input type="checkbox"/> Scale         | <input type="checkbox"/> Reference Frame    | <input checked="" type="checkbox"/> Representation |
| <input type="checkbox"/> Continuity    | <input type="checkbox"/> Boundedness        | <input type="checkbox"/> Invariance/Symmetry       |
| <input type="checkbox"/> Equivalence   | <input type="checkbox"/> General/Particular | <input type="checkbox"/> Contradiction             |
| <input type="checkbox"/> Use of Limits | <input type="checkbox"/> Approximation      | <input type="checkbox"/> Other                     |

1. Each row of the table has consecutive numbers in it. Also, each column has consecutive numbers in it. The table can be filled out simply by increasing the numbers by one as you move down from the first row.

The numbers also represent an addition table from 1 to 9, i.e., the number in each square is the sum of the row number (black) and the column number (gray).

2.

- a. The number in a square with top number 7 and side number 8 is 15.
- b. Here it becomes important to notice that the number in each square is the sum of the top number and the side number. The number in the square is 50.
- c. To find the number in the square, it is most efficient to add the top number and the side number. However, it is possible to approach this question in slightly different ways. For example, instead of giving a formulaic definition (top number plus side number), a student might give a procedural definition: for example, one could say that the first number in a row is one more than the side number, or the second number in a row is two more than the row number, etc. Then, for instance, the 20<sup>th</sup> number in a row, i.e., the number in a cell with top number 20, is 20 more than the side number. While this definition may appear very different from the formulaic definition, the two approaches are functionally equivalent.

3. The answer, in general, is “No.” The same number may appear in several consecutive rows. For example, number 5 appears in the first, second, third and fourth rows. So it would not be possible to tell what the side number and the top numbers are for a square with 5 in it.

- 4.**
- a.** The number is 5. It can either be computed (see below), or found directly from the table.
  - b.** Here it becomes more difficult, once again, to find the number by manipulating the table directly. The top number is 32.
  - c–d.** Since the number in the square is the sum of the side number and the top number, if one of these numbers is known, the other is the difference between the number in the square and the known number. It is imperative for students to understand that the process of finding the top or the side number from the number in the square and the other known number is the *inverse* of the process of finding the number in the square from the top and the side numbers.
- 5.** If the top and the side numbers are the same, the number in the square is the sum of two equal numbers, or, simply, a double of either the side or the top number. So, to find the number in the square, simply double one of the given numbers.
- 6.** The table should show a checkerboard pattern.

	<b>partial level (1 or 2)</b>	<b>full level (3)</b>
<b>Modeling/ Formulating (weight: 2)</b>	Student applies procedural rules throughout, and does not form abstractions from the rules.	Student is able to state general rules, with only indirect reference to the search process.
<b>Transforming/ Manipulating (weight: 2)</b>	Student completes some of the computations correctly and fills out the table according to a specific rule, with a few minor computational errors. Shows only a partial pattern in question 6.	Student completes all the computations correctly, and fills out the table completely according to a single specific rule, even if this rule is not correct. Shows complete checkerboard pattern in 6.
<b>Inferring/ Drawing Conclusions (weight: 3)</b>	Student completes the table and computes the values using only the consecutive number patterns.	Student completes the table correctly and draw general conclusions about the rules for finding numbers; also uses these rules to find the numbers <i>outside</i> of the given table. Correctly determines the even and odd numbers in question 6.
<b>Communicating (weight: 2)</b>	Student completes some of the table and/or gives incomplete reasoning for other answers.	Student completes the table and answers each question about general rules completely. Shows the complete pattern in question 6.